...A
Quadratics

Name KEY

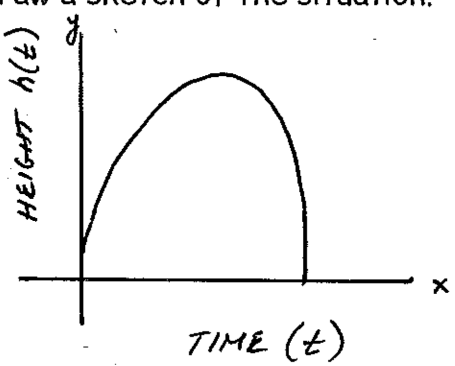
Learning Target: I can analyze and model quadratic functions.

Fireworks

A firework is really a rocket that is shot up into the air and explodes really high in the air. What happens to the rocket when it explodes? Loud pop and colorful lights are the result! Once it explodes, debris falls to the ground from the firework. There is a function h(t) that will give the rocket's height off the ground in terms of the time t elapsed since launch. Specifically, if t is in seconds and h(t) is in feet, then $h(t) = 160 + 92t - 16t^2$

Based on this information, please answer the questions below.

1. Draw a sketch of the situation.



2. What is the height of the rocket after 2 seconds? After 2.5 seconds? After 3 seconds?

$h(2) = -16(2)^2 + 92(2) + 160$

$\boxed{h(2) = 280 \text{ FT}}$

$h(2.5) = -16(2.5)^2 + 92(2.5) + 160$

$\boxed{h(2.5) = 290 \text{ FT}}$

$h(3) = -16(3)^2 + 92(3) + 160$

$\boxed{h(3) = 292 \text{ FT}}$

3. How high does the rocket get?

$X = \frac{-B}{2A} = \frac{-92}{2(-16)} = \frac{-92}{-32} = 2.875$ SECONDS

$h(2.875) = -16(2.875)^2 + 92(2.875) + 160$

$\boxed{h(2.875) = 292.25 \text{ FT}}$

4. How long does it take for the rocket to reach its highest point?

$$x = \frac{-B}{2A} = \frac{-92}{2(-16)} = \frac{-92}{-32} = \boxed{2.875 \text{ SECONDS}}$$

5. How many seconds does it take for the rocket to be 250 feet high?

$$h(t) = -16t^2 + 92t + 160$$

$$250 = -16t^2 + 92t + 160$$

$$0 = -16t^2 + 92t - 90$$

$$x = \frac{-92 \pm \sqrt{(92)^2 - 4(-16)(-90)}}{2(-16)}$$

$$x = \frac{-92 \pm \sqrt{2704}}{-32}$$

$$x = \frac{-92 \pm 52}{-32}$$

$$\boxed{x = 1.25 \text{ SEC}} \quad \boxed{x = 4.5 \text{ SEC}}$$

6. When does the debris from the rocket hit the ground?

$$h(t) = -16t^2 + 92t + 160$$

$$0 = -16t^2 + 92t + 160$$

$$x = \frac{-92 \pm \sqrt{(92)^2 - 4(-16)(160)}}{2(-16)}$$

$$x = \frac{-92 \pm \sqrt{18704}}{-32}$$

$$x = -1.399 \text{ SEC}$$

$$\boxed{x = 7.149 \text{ SEC}}$$

7. Based on the graph of $h(t) = 160 + 92t - 16t^2$, is the rocket being launched off of the ground or off of a building? Please explain how you know.

OFF OF A BUILDING. THE INTIAL HEIGHT OF THE ROCKET IS 160 FT.

$$h(0) = -16(0)^2 + 92(0) + 160$$

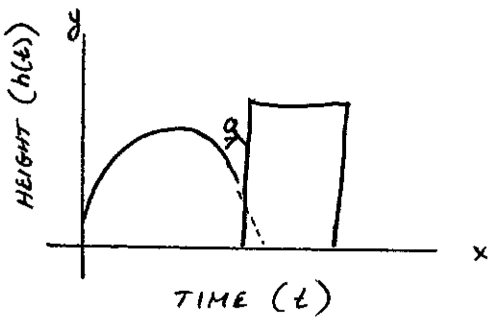
$$h(0) = 160 \text{ FT}$$

Keys

Miranda throws a set of keys up to her brother, who is standing on a third-story balcony with his hands 38 feet above the ground. If Miranda throws the keys with an initial velocity of 40 feet her second, the equation $h = -16t^2 + 40t + 5$ gives the height of the keys after t seconds.

Based on this information, please answer the questions below.

1. Draw a sketch of the situation.



2. How long does it take the keys to reach their highest point?

$$x = \frac{-B}{2A} = \frac{-40}{2(-16)} = \frac{-40}{-32} = \boxed{1.25 \text{ SEC}}$$

$$h(1.25) = -16(1.25)^2 + 40(1.25) + 5$$

$$h(1.25) = 30 \text{ FT}$$

3. How high do the keys reach?

$$\boxed{30 \text{ FT}}$$

4. Will her brother be able to catch the keys? Explain.

NO. THE MAX HEIGHT REACHED WAS 30 FT. NEEDED TO REACH 38 FT.

To the Rescue

A helicopter is flying to drop a supply bundle to a group of firefighters who are behind the fire lines. At the moment when the helicopter crew makes the drop, the helicopter is hovering 400 feet above the ground. The principles of physics that describe the behavior of falling objects state that when an object is falling freely, it goes faster and faster as it falls. In fact, these principles provide a specific formula describing the object's fall, which can be expressed this way:

Suppose that the object's height off the ground when it begins to fall, at time $t = 0$, is N feet, and use $h(t)$ to represent the object's height off the ground t seconds after being dropped. Then the function $h(t)$ is given by the equation $h(t) = N - 16t^2$. So in the case of the falling supplies, the formula is $h(t) = 400 - 16t^2$, because the supply bundle is 400 feet off the ground when it starts to fall.

1. How many seconds will it take the bundle to reach the ground?

$0 = 400 - 16t^2$

$16t^2 = 400$

$t^2 = 25$

$t = \pm 5$

$\boxed{t = 5 \text{ SEC}}$

2. Write an equation that you could use to find out how many seconds it takes until the supply bundle is 100 feet off the ground.

$h(t) = 400 - 16t^2$

$100 = 400 - 16t^2$

$\boxed{0 = 300 - 16t^2}$

3. Use a graphing calculator to find an approximate solution to your equation from Question 2.

$\boxed{t = 4.33 \text{ SECONDS}}$

4. Explain how you could check your solution from Question 2 using the formula $h(t) = 400 - 16t^2$.

EVALUATE $h(t)$ WHEN $t = 4.33$

Submarine

A submarine is an enclosed ship that can dive under water and reach deep depths of the ocean. The submarine Big Blue went on a trial run and the function $f(x) = x^2 - 9x$ models this trial run where x represents the number of minutes for the trial run and f(x) represents the depth of the submarine in yards.

1. When graphing $f(x) = x^2 - 9x$, what do the x and y represent?

$x \rightarrow$ # OF MINUTES FOR THE TRIAL RUN

$y \rightarrow$ DEPTH

2. What is the deepest depth the submarine will go?

$x = \frac{-(-9)}{2(1)} = \frac{9}{2} = 4.5$

$f(x) = x^2 - 9x$

$f(4.5) = (4.5)^2 - 9(4.5)$

$f(4.5) = \boxed{-20.25 \text{ YARDS}}$

3. What is the depth of the submarine at 8 seconds?

$f(8) = (8)^2 - 9(8)$

$\boxed{f(8) = -8 \text{ YARDS}}$

4. How long did the trial run last?

$0 = x^2 - 9x$

$0 = x(x-9)$

$x = 0 \quad x - 9 = 0$

$x = 9$

$\boxed{9 \text{ MIN}}$

5. When will the submarine get to its deepest depth?

$\boxed{4.5 \text{ MIN}}$

6. How many minutes does it take for the submarine to get 16 yards below sea level?

$f(x) = x^2 - 9x$

$-16 = x^2 - 9x$

$\boxed{0 = x^2 - 9x + 16}$

Projectile

The height in feet of a projectile with an initial velocity of 64 feet per second and an initial height of 80 feet is a function of time t in second given by $h(t) = -16t^2 + 64t + 80$.

1. Find the maximum height of the projectile.

$h(2) = -16(2)^2 + 64(2) + 80$

$\boxed{h(2) = 144 \text{ FT}}$

2. Find the time t when the projectile achieves its maximum height.

$x = \frac{-B}{2A} = \frac{-64}{2(-16)} = \frac{-64}{-32} = \boxed{2 \text{ SEC}}$

3. Find the time t when the projectile hits the ground.

$0 = -16t^2 + 64t + 80$

$0 = -16(t^2 - 4t - 5)$

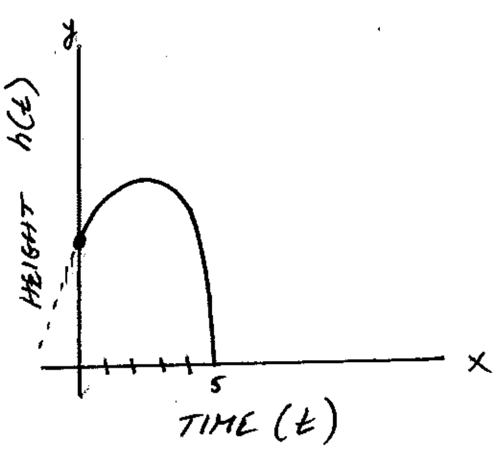
$0 = -16(t-5)(t+1)$

$t - 5 = 0 \quad t + 1 = 0$

$t = 5 \quad t = -1$

$\boxed{5 \text{ SECONDS}}$

4. Sketch the graph of the projectile.



Business

Kelly owns a business making decorative boxes to store jewelry, mementos, and other valuables. The function $y = x^2 + 50x + 1800$ models the profit y that Kelly has made in month x for the first two years of her business.

1. Write an equation representing the month in which Kelly's profit is $2400.

$y = x^2 + 50x + 1800$

$2400 = x^2 + 50x + 1800$

$\boxed{0 = x^2 + 50x - 600}$

$0 = (x+60)(x-10)$

$x + 60 = 0$ $\quad$ $x - 10 = 0$

$x = -60$ $\quad$ $x = 10$

2. Solve to find out which month Kelly's profit is $2400.

$\boxed{\text{10 MONTHS}}$

3. What is her profit in month 12?

$y = x^2 + 50x + 1800$

$y = (12)^2 + 50(12) + 1800$

$y = 2544$

$\boxed{\$2544}$

4. What is her profit in month 24?

$y = x^2 + 50x + 1800$

$y = (24)^2 + 50(24) + 1800$

$y = 3576$

$\boxed{\$3576}$

5. Did her profit double between months 12 and 24? Explain.

NO. THE EQUATION IS NOT LINEAR.

Roofer

A roofer tosses a piece of roofing tile from a roof onto the ground 30 feet below. He tosses the tile with an initial downward velocity of 10 feet per second.

Write an equation to find how long it takes the tile to hit the ground. Use the model for vertical motion, $H = -16t^2 + vt + h$, where H is the height of an object after t seconds, v is the initial velocity, and h is the initial height.

$$H = -16t^2 + 10t + 30$$

How long does it take the tile to hit the ground?

$$0 = -16t^2 + 10t + 30$$

$$x = \frac{-10 \pm \sqrt{(10)^2 - 4(-16)(30)}}{2(-16)}$$

$$x = \frac{-10 \pm \sqrt{2020}}{-32}$$

$$x = -1.092 \qquad \boxed{x = 1.717 \text{ SEC}}$$

Ball

Jimmy tosses a ball up to Tommy, waiting at a third-story window, with an initial velocity of 30 feet per second. He releases the ball from a height of 6 feet. The equation $h = -16t^2 + 30t + 6$ represents the height h of the ball after t seconds. If the ball must reach a height of 25 feet for Tommy to catch it, does the ball reach Tommy?

$$x = \frac{-B}{2A} = \frac{-30}{2(-16)} = \frac{-30}{-32} = 0.9375$$

$$h = -16(0.9375)^2 + 30(0.9375) + 6$$

$$h = 20.0625 \text{ FT}$$

NO, THE BALL WILL NOT REACH TOMMY BECAUSE THE MAX HEIGHT OF THE BALL IS 20 FT. IT NEEDED TO REACH A HEIGHT OF 25 FT.