

MATRICES

A Matrix is simplified version of working with equations with multiple variables.

If a car company is building cars and trucks they can use matrices to determine the number of parts they will need over a given span of time, producing a particular number of vehicles. If each car needs 4 wheels, 2 bench seats, and 1 gas tank. Each truck needs 6 wheels, 1 bench seat and 3 gas tanks. Then we can set-up a matrix where each row and column are for a given part of the equation.

$$\begin{matrix} & \begin{matrix} w & s & g \end{matrix} \\ \begin{matrix} c \\ t \end{matrix} & \begin{bmatrix} 4 & 2 & 1 \\ 6 & 1 & 3 \end{bmatrix} \end{matrix}, \text{ where the } c=\text{cars, } t=\text{trucks, } w=\text{wheels, } s=\text{seats, and } g=\text{gas tanks}$$

Using matrices we can solve for all kinds of situations. Matrices have their own specific rules for adding, subtracting, multiplying, and dividing.

The size (dimension) of a Matrix is # Rows by # Columns. (Rows go across, columns up and down)

EXAMPLE: $B = \begin{bmatrix} 3 & 2 \\ 1 & 0 \\ -1 & -2 \end{bmatrix}$ ← row

Matrix B is a 3 x 2 matrix.

↑ Column

An element of a Matrix is the value in a particular position.

EXAMPLE: $B = \begin{bmatrix} 3 & 2 \\ 1 & 0 \\ -4 & -5 \end{bmatrix} b_{row, column} \quad b_{1,2} = 2$

2 is the element in the 1st row and 2nd column

Use the matrices below to answer all questions.

$$A = \begin{bmatrix} 1 & 0 & -2 \\ 2 & 3 & 5 \\ 2 & -3 & 0 \end{bmatrix} \quad B = \begin{bmatrix} 3 & 2 \\ 1 & 0 \\ -1 & -2 \end{bmatrix} \quad C = \begin{bmatrix} -3 & 0 & 2 \\ 1 & -1 & 0 \\ 0 & -4 & 3 \end{bmatrix} \quad D = \begin{bmatrix} -2 & -2 \\ 7 & 9 \\ 3 & 6 \end{bmatrix}$$

$$E = [2 \quad -8 \quad 13 \quad 5] \quad F = \begin{bmatrix} 4 \\ 7 \end{bmatrix} \quad G = \begin{bmatrix} 0 & 2 & -4 \\ 3 & 5 & -5 \\ 1 & 1 & 6 \end{bmatrix} \quad H = \begin{bmatrix} -4 & 2 & 1 & 0 \\ -2 & -1 & 4 & 1 \end{bmatrix}$$

List the dimensions for the specified matrix

- 1. E 1 x 4
- 2. F 2 x 1
- 3. D 3 x 2
- 4. H 2 x 4
- 5. A 3 x 3
- 6. B 3 x 2

Identify the element in the specified locations, If possible.

5. Matrix D, $d_{2,1}$
row, column

5. 7

6. Matrix A, $a_{2,3}$

6. 5

7. Matrix H, $h_{4,1}$

7. 0

8. Matrix E, $e_{1,3}$

8. 13

If the Matrices are set equal to each other, *each element must be the same.*

Solve for all variables

$$9. \begin{bmatrix} 4 & x \\ y+3 & -8 \end{bmatrix} = \begin{bmatrix} 4 & 0 \\ 12 & z-8 \end{bmatrix}$$

$$x=0$$

$$y+3=12$$

$$y=9$$

$$-8=z-8$$

$$0=z$$

$$10. \begin{bmatrix} 2a+1 & 16 \\ 7-b & 1 \end{bmatrix} = \begin{bmatrix} 17 & 16 \\ -15 & c+4 \end{bmatrix}$$

$$2a+1=17$$

$$2a=16$$

$$a=8$$

$$7-b=-15$$

$$-b=-22$$

$$b=22$$

$$1=c+4$$

$$-3=c$$

ADDING, SUBTRACTING, AND SCALAR MULTIPLICATION

When Adding and Subtracting Matrices, the matrices *must be the same exact size!*

Adding- make sure you add ALL elements in the 2nd matrix.

Subtracting – make sure you subtract ALL elements in the 2nd matrix.

Scalar Multiplication – make sure you distribute the multiplier to ALL elements in the matrix.

EXAMPLES: $A = \begin{bmatrix} 1 & 0 & -2 \\ 2 & 3 & 5 \\ 2 & -3 & 0 \end{bmatrix}$ $C = \begin{bmatrix} -3 & 0 & 2 \\ 1 & -1 & 0 \\ 0 & -4 & 3 \end{bmatrix}$ Use the following matrices for these examples:

Work:

$$1. A+C = \begin{bmatrix} 1+(-3) & 0+0 & -2+2 \\ 2+1 & 3+(-1) & 5+0 \\ 2+0 & -3+(-4) & 0+3 \end{bmatrix} \quad 2. A-C = \begin{bmatrix} 1-(-3) & 0-0 & -2-2 \\ 2-1 & 3-(-1) & 5-0 \\ 2-0 & -3-(-4) & 0-3 \end{bmatrix} \quad 3. 4A = \begin{bmatrix} 4(1) & 4(0) & 4(-2) \\ 4(2) & 4(3) & 4(5) \\ 4(2) & 4(-3) & 4(0) \end{bmatrix}$$

Answer:

$$1. A+C = \begin{bmatrix} -2 & 0 & 0 \\ 3 & 2 & 5 \\ 2 & -7 & 3 \end{bmatrix} \quad 2. A-C = \begin{bmatrix} 4 & 0 & -4 \\ 1 & 4 & 5 \\ 2 & 1 & -3 \end{bmatrix} \quad 3. 4A = \begin{bmatrix} 4 & 0 & -8 \\ 8 & 12 & 20 \\ 8 & -12 & 0 \end{bmatrix}$$

Perform the appropriate operation on the given matrices. SHOW ALL YOUR WORK!!

$$A = \begin{bmatrix} 1 & 0 & -2 \\ 2 & 3 & 5 \\ 2 & -3 & 0 \end{bmatrix} \quad C = \begin{bmatrix} -3 & 0 & 2 \\ 1 & -1 & 0 \\ 0 & -4 & 3 \end{bmatrix} \quad D = \begin{bmatrix} -2 & -2 \\ 7 & 9 \\ 3 & 6 \end{bmatrix} \quad B = \begin{bmatrix} 3 & 2 \\ 1 & 0 \\ -1 & -2 \end{bmatrix} \quad G = \begin{bmatrix} 0 & 2 & -4 \\ 3 & 5 & -5 \\ 1 & 1 & 6 \end{bmatrix}$$

$$11. D + B \quad \begin{bmatrix} -2 & -2 \\ 7 & 9 \\ 3 & 6 \end{bmatrix} + \begin{bmatrix} 3 & 2 \\ 1 & 0 \\ -1 & -2 \end{bmatrix} = \begin{bmatrix} -2+3 & -2+2 \\ 7+1 & 9+0 \\ 3+(-1) & 6+(-2) \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 8 & 9 \\ 2 & 4 \end{bmatrix}$$

$$12. G - C \quad \begin{bmatrix} 0 & 2 & -4 \\ 3 & 5 & -5 \\ 1 & 1 & 6 \end{bmatrix} - \begin{bmatrix} -3 & 0 & 2 \\ 1 & -1 & 0 \\ 0 & -4 & 3 \end{bmatrix} = \begin{bmatrix} 0-(-3) & 2-0 & -4-2 \\ 3-1 & 5-(-1) & -5-0 \\ 1-0 & 1-(-4) & 6-3 \end{bmatrix} = \begin{bmatrix} 3 & 2 & -6 \\ 2 & 6 & -5 \\ 1 & 5 & 3 \end{bmatrix}$$

$$13. 3B \quad 3 \begin{bmatrix} 3 & 2 \\ 1 & 0 \\ -1 & -2 \end{bmatrix} = \begin{bmatrix} 3(3) & 3(2) \\ 3(1) & 3(0) \\ 3(-1) & 3(-2) \end{bmatrix} = \begin{bmatrix} 9 & 6 \\ 3 & 0 \\ -3 & -6 \end{bmatrix}$$

14. $G + A - C$

$$\begin{bmatrix} 0 & 2 & -4 \\ 3 & 5 & -5 \\ 1 & 1 & 6 \end{bmatrix} + \begin{bmatrix} 1 & 0 & -2 \\ 2 & 3 & 5 \\ 2 & -3 & 0 \end{bmatrix} - \begin{bmatrix} -3 & 0 & 2 \\ 1 & -1 & 0 \\ 0 & -4 & 3 \end{bmatrix} = \begin{bmatrix} 0+1-(-3) & 2+0-0 & -4+(-2)-2 \\ 3+2-1 & 5+3-(-1) & -5+5-0 \\ 1+2-0 & 1+(-3)-(-4) & 6+0-3 \end{bmatrix} = \begin{bmatrix} 4 & 2 & -8 \\ 6 & 9 & 0 \\ 3 & 6 & 3 \end{bmatrix}$$

15. $4D + -3B$

$$4 \begin{bmatrix} -2 & -2 \\ 7 & 9 \\ 3 & 6 \end{bmatrix} + -3 \begin{bmatrix} 3 & 2 \\ 1 & 0 \\ -1 & -2 \end{bmatrix} = \begin{bmatrix} 4(-2) & 4(-2) \\ 4(7) & 4(9) \\ 4(3) & 4(6) \end{bmatrix} + \begin{bmatrix} -3(3) & -3(2) \\ -3(1) & -3(0) \\ -3(-1) & -3(-2) \end{bmatrix}$$

$$\begin{bmatrix} -8 & -8 \\ 28 & 63 \\ 12 & 24 \end{bmatrix} + \begin{bmatrix} -9 & -6 \\ -3 & 0 \\ 3 & 6 \end{bmatrix} = \begin{bmatrix} -8+(-9) & -8+(-6) \\ 28+(-3) & 63+0 \\ 12+3 & 24+6 \end{bmatrix} = \begin{bmatrix} -17 & -14 \\ 25 & 63 \\ 15 & 30 \end{bmatrix}$$